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# PREDICTION OF IN-FLIGHT FUNDAMENTAL MODE OF LAUNCH VEHICLE STRUCTURE FROM TWO POINT GROUND VIBRATION TESTS

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## 1. INTRODUCTION

Ground Resonance Tests (GRT) play an important role in the development of flight vehicles inasmuch as that these tests provide the modal information for both elastodynamic as well as flight control related studies for the vehicle. It is, therefore, necessary to obtain from the ground tests information which is as accurate as possible regarding the elastic modes of the structure, of which the fundamental mode is the most important. In general, ground tests [1, 2] suffer from the basic handicap that the flight structure needs to be supported in some way for the tests, while providing as close to the free–free boundary condition as possible [3]. These two contradictory requirements have led to the most universally acceptable configuration for the launch vehicle ground test, in which the structure is horizontally supported on two points corresponding to the two nodes of the theoretically estimated fundamental mode, using thin wires, hanging vertically. In general, as the nodes of the fundamental mode are only approximately known from a theoretical analysis, there is usually a small error in spanwise positioning of the support points, resulting from various factors including differences arising from mismatches in structural stiffness and inertia distributions of the fabricated vehicle, and it is necessary to assess the sensitivity of the fundamental mode frequency and generalized modal mass to such errors. In the literature, no studies have been encountered that have considered the sensitivity of the in-flight fundamental mode to positioning errors in two point support in the ground vibration test, and the present study is a modest attempt to examine this sensitivity and to evolve a simple empirical methodology for predicting the in-flight fundamental mode from ground vibration tests with greater confidence.

## 2. FORMULATION AND SOLUTION

The transverse vibration of the launch vehicle structures, modelled as a multi-segment stepped beam, which is assumed to be fairly slender, can be adequately described using the Euler–Bernoulli beam theory [4] with a set of $N$ partial differential equations having constant coefficients as follows:

$$(\partial^4 \bar{w}_i / \partial \bar{x}_i^4) + \gamma_i^4 \bar{w}_i = 0, \tag{1}$$

where $i$ $(=1, \ldots, N)$ is the constant property segment identifier and $\gamma_i^4 (q = [\ (\rho A)_i \omega^2 L_i^4)$ is the dimensionless frequency parameter of the $i$th launch vehicle segment. The span co-ordinate $\bar{x}_i (= x_i / L_0)$ refers to the $i$th segment and takes values from 0 to $L_i / L_0$ in each of the $N$ segments. A new frequency parameter for the complete launch vehicle structure can be defined as,

$$\lambda^4 = \rho A_0 \omega^2 L_0^4 / EI_0, \tag{2}$$

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where $\rho A_0$, $L_0$ and $EI_0$ are the reference values, which can be chosen as convenient. The general solution of equation (1) can be written as

$$\bar{w}_i = A_i \cosh \gamma_i \bar{x}_i + B_i \sinh \gamma_i \bar{x}_i + C_i \cos \gamma_i \bar{x}_i + D_i \sin \gamma_i \bar{x}_i, \quad (3)$$

where $A_i$, $B_i$, $C_i$ and $D_i$ are arbitrary constants of integration. The above solution represents $4N$ unknown constants that are to be determined by applying four free–free boundary conditions at the two free ends of the structure and $4(N-1)$ conditions of continuity on the displacement, slope, shear force and bending moment at the junction of two segments. These free end boundary conditions are as follows:

$$\bar{w}_1'''(0) = \bar{w}_1''(0) = 0, \qquad \bar{w}_N'''(\bar{L}_N) = \bar{w}_N''(\bar{L}_N) = 0. \quad (4,\ 5)$$

The configuration of the two point supports for the launch vehicle structure is taken as a combination of a very stiff linear spring (non-dimensional constant $K_l$) and a very soft rotational spring (non-dimensional constant $K_r$) at two junctions that are fairly close to the actual nodal points of the fundamental mode. The continuity conditions at junctions that are defined as support points [5] are as follows:

$$\bar{w}_i(\bar{L}_i) = \bar{w}_{i+1}(0), \qquad \bar{w}_i'(\bar{L}_i) = \bar{w}_{i+1}'(0), \quad (6,\ 7)$$

$$\bar{w}_i''(\bar{L}_i) = \bar{w}_{i+1}''(0) + K_r \bar{w}_{i+1}'(0), \qquad \bar{w}_i'''(\bar{L}_i) = \bar{w}_{i+1}'''(0) + K_l \bar{w}_{i+1}'(0). \quad (8,\ 9)$$

The shear force and bending moment continuity conditions at other junctions are as follows, while the displacement and slope continuity conditions remain as in equations (6) and (7):

$$\bar{w}_i''(\bar{L}_i) = \bar{w}_{i+1}''(0), \qquad \bar{w}_i'''(\bar{L}_i) = \bar{w}_{i+1}'''(0). \quad (10,\ 11)$$

Satisfaction of these $4N$ conditions leaves only the frequency parameter $\lambda$ as an unknown. This is obtained by setting the characteristic determinant to zero. The corresponding mode shape is obtained by substituting this value of the frequency parameter into the simultaneous equations involving the unknown constants $A_i$, $B_i$, $C_i$ and $D_i$. It may be mentioned here that the zeroes of the characteristic determinant are obtained through a

TABLE 1

*The structural geometry of the generic launch vehicle* [6]

| Segment | Mass (kg) | Length (m) | *EI* ($Nm^2$) |
|---|---|---|---|
| 1 | 2131 | 1·693 | $2·35 \times 10^8$ |
| 2 | 5048 | 3·022 | $3·35 \times 10^8$ |
| 3 | 8900 | 5·328 | $3·35 \times 10^8$ |
| 4 | 146 | 0·110 | $3·00 \times 10^8$ |
| 5 | 850 | 1·860 | $1·50 \times 10^8$ |
| 6 | 3189 | 3·932 | $1·30 \times 10^8$ |
| 7 | 259 | 0·310 | $1·30 \times 10^8$ |
| 8 | 65 | 0·748 | $1·50 \times 10^8$ |
| 9 | 398 | 0·500 | $0·77 \times 10^8$ |
| 10 | 311 | 0·500 | $2·40 \times 10^8$ |
| 11 | 621 | 1·000 | $0·31 \times 10^8$ |
| 12 | 218 | 1·000 | $0·85 \times 10^8$ |
| 13 | 265 | 1·000 | $0·60 \times 10^8$ |
| 14 | 286 | 1·000 | $0·53 \times 10^8$ |
| 15 | 135 | 0·560 | $0·90 \times 10^8$ |
| 16 | 7 | 0·940 | $0·49 \times 10^8$ |

TABLE 2

*Single support point shift error sensitivity*

| | $K_r = 0{\cdot}00$ | | $K_r = 0{\cdot}02$ | | $K_r = 0{\cdot}05$ | |
|---|---|---|---|---|---|---|
| Error, $e$ | Frequency, $\lambda$ | Mass, $m$ | Frequency, $\lambda$ | Mass, $m$ | Frequency, $\lambda$ | Mass, $m$ |
| −0·05 | 4·609 | 0·139 | 4·604 | 0·139 | 4·597 | 0·139 |
| −0·04 | 4·653 | 0·151 | 4·647 | 0·151 | 4·640 | 0·151 |
| −0·03 | 4·687 | 0·167 | 4·682 | 0·167 | 4·674 | 0·167 |
| −0·02 | 4·711 | 0·189 | 4·706 | 0·189 | 4.697 | 0.189 |
| −0·01 | 4·726 | 0·216 | 4·720 | 0·216 | 4·711 | 0·216 |
| 0·00 | 4·730 | 0·250 | 4·724 | 0·250 | 4·715 | 0·250 |
| 0·01 | 4·726 | 0·239 | 4·720 | 0·239 | 4·711 | 0·239 |
| 0·02 | 4·713 | 0·232 | 4·707 | 0·232 | 4·698 | 0·232 |
| 0·03 | 4·694 | 0·228 | 4·688 | 0.228 | 4.679 | 0·228 |
| 0·04 | 4·670 | 0·227 | 4·664 | 0·227 | 4·654 | 0·227 |
| 0·05 | 4·640 | 0·228 | 4·634 | 0·228 | 4·625 | 0·228 |

TABLE 3

*Both support point symmetric shift error sensitivity*

| | $K_r = 0{\cdot}00$ | | $K_r = 0{\cdot}02$ | | $K_r = 0{\cdot}05$ | |
|---|---|---|---|---|---|---|
| Error, $e$ | Frequency, $\lambda$ | Mass, $m$ | Frequency, $\lambda$ | Mass, $m$ | Frequency, $\lambda$ | Mass, $m$ |
| −0·05 | 4·574 | 0·198 | 4·569 | 0·198 | 4·563 | 0·198 |
| −0·04 | 4·624 | 0·201 | 4·619 | 0·201 | 4·612 | 0·201 |
| −0·03 | 4·667 | 0·207 | 4·662 | 0·207 | 4·655 | 0·207 |
| −0·02 | 4·701 | 0·216 | 4·696 | 0·216 | 4·685 | 0·216 |
| −0·01 | 4·723 | 0·230 | 4·717 | 0·230 | 4·708 | 0·230 |
| 0·00 | 4·730 | 0·250 | 4·724 | 0·250 | 4·715 | 0·250 |
| 0·01 | 4·722 | 0·277 | 4·716 | 0·277 | 4·706 | 0·277 |
| 0·02 | 4·697 | 0·314 | 4·691 | 0·314 | 4·681 | 0·313 |
| 0·03 | 4·657 | 0·362 | 4·651 | 0·362 | 4·641 | 0·362 |
| 0·04 | 4·604 | 0·425 | 4·598 | 0·424 | 4·587 | 0·424 |
| 0·05 | 4·538 | 0·428 | 4·532 | 0·427 | 4·522 | 0·428 |

combination of Regula-falsi slope search method and interval halving technique and roots are extracted with a double precision accuracy of $1{\cdot}0 \times 10^{-6}$.

## 3. RESULTS FOR TWO POINT SUPPORTED UNIFORM BEAM

A uniform beam of unit length, mass and bending rigidity is undertaken for the study in order to establish the solution sensitivity for a generic non-dimensional launch vehicle structure, with a linear spring constant of $10^{10}$. In Tables 2–4 are presented the results for the frequency parameter $\lambda$ and the modal mass parameter $m$, as a function of the non-dimensional error in the support location, $e$, for three different cases of the support error configuration, as (1) single support point error, (2) both support point symmetric error, and (3) both support point sideways error, and for three different values of the rotational spring constant, as 0·00, 0·02 and 0·05, under the assumption that rotational restraint of the support is a fairly small fraction of the launch vehicle rotational stiffness. The support location error parameter is taken to be zero for the case when the support

TABLE 4

*Both support point sideways shift error sensitivity*

| | $K_r = 0{\cdot}00$ | | $K_r = 0{\cdot}02$ | | $K_r = 0{\cdot}05$ | |
|---|---|---|---|---|---|---|
| Error, e | Frequency, $\lambda$ | Mass, $m$ | Frequency, $\lambda$ | Mass, $m$ | Frequency, $\lambda$ | Mass, $m$ |
| −0·05 | 4·520 | 0·135 | 4·515 | 0·135 | 4·507 | 0·135 |
| −0·04 | 4·588 | 0·144 | 4·583 | 0·144 | 4.575 | 0.144 |
| −0·03 | 4·647 | 0·157 | 4·641 | 0·157 | 4·633 | 0·158 |
| −0·02 | 4·692 | 0·177 | 4·686 | 0·178 | 4·677 | 0·178 |
| −0·01 | 4·720 | 0·207 | 4·714 | 0·207 | 4.705 | 0.208 |
| 0·00 | 4·730 | 0·205 | 4·724 | 0·250 | 4·715 | 0·250 |

coincides with the theoretical nodal point location, is negative (−ve) when the support is to the left of nodal point and is positive (+ve) when support is to the right of nodal point. The results in Tables 2 and 3 show that the frequency parameter $\lambda$ is nearly symmetric with respect to both +ve and −ve support errors around zero error point for all the three values of the rotational restraint parameter $K_r$, even though the sensitivity is slightly more when the two supports move away from each other in comparison to the case when these two move closer. This can be attributed to the fact that curvature changes are more pronounced when the supports move away from each other. This fact is further brought out by the significant changes that take place in the values of $m$ with both +ve and −ve shift in the support location.

In particular, Table 3 shows that a net 10% inward movement of the support points leads to a 3·3% reduction in $\lambda$ (or a 6·5% reduction in the frequency $\omega$) and a 20·8% reduction in modal mass $m$. These values for a net 10% outward movement are a 4·1% reduction in $\lambda$ and a 71% increase in $m$, respectively, and they clearly show that even a small shift in the support point location can introduce significant errors in the frequency measurement. The effect of small rotational restraint at the support point, as seen from Tables 2 and 3, is marginal on the frequency parameter $\lambda$ and nearly non-existent on the modal mass parameter $m$, in that a 5% restraint at both the support points causes about

TABLE 5

*One support point sideways shift error sensitivity for typical launch vehicle* [6]: $EI_0 = 2{\cdot}35 \times 10^8$, $\rho A_0 = 971{\cdot}3$, *total mass* = 22 829, *length* = 23·503

| Error, $e$ | Frequency parameter, $\lambda$ | Cyclic frequency, $f$ (Hz) | Modal mass, $m$ (kg) |
|---|---|---|---|
| −0·06 | 4·867 | 3·331 | 1040·9 |
| −0·05 | 4·904 | 3·382 | 1007·0 |
| −0·04 | 4·936 | 3·427 | 991·1 |
| −0·03 | 4·964 | 3·465 | 984·4 |
| −0·02 | 4·985 | 3·494 | 979·8 |
| −0·01 | 4·998 | 3·513 | 975·3 |
| 0·00 | 5·003 | 3·502 | 995·7 |
| 0·01 | 4·997 | 3·512 | 1063·6 |
| 0·02 | 4·980 | 3·488 | 1154·0 |
| 0·03 | 4·950 | 3·446 | 1289·8 |
| 0·04 | 4·907 | 3·386 | 1493·6 |
| 0·05 | 4·851 | 3·309 | 1765·0 |
| 0·06 | 4·783 | 3·217 | 2127·0 |

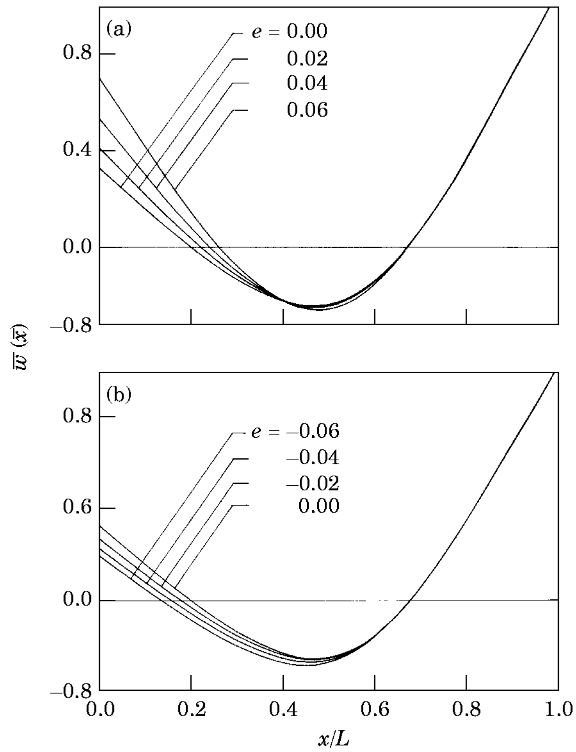


Figure 1. The normalized fundamental mode of the generic launch vehicle for various values of (a) +ve support error and (b) −ve support error at the first support point.

a 0·4% reduction in $\lambda$ (or a 0·9% reduction in frequency). In Table 4 are presented the results for the uniform beam when error in both the support location is on one side, so that the effects are additive, and it can be seen that a net 5% sideways shift causes a 4·4% reduction in $\lambda$ (or a 9% reduction in frequency) and a 46% reduction in modal mass $m$, indicating an increased sensitivity to such a uniform shift. It can be seen that any error configuration can be reconstructed as a combination of the three cases that have been presented in Tables 2–4.

## 4. RESULTS FOR TWO POINT SUPPORTED GENERIC LAUNCH VEHICLE

In Table 5 are presented the results for frequency parameter, $\lambda$, cyclic frequency, $f$ (in Hz) and modal mass $m$ (in kg) for a typical launch vehicle structure from reference [6] with a generic structural configuration as given in Table 1, for the case in which the first support point moves in both the +ve and −ve directions around the correct nodal point of the free–free vehicle.

The linear support in this case is also taken to be nearly rigid ($K_1 = 10^{10}$), with zero rotational restraint. It may be noted here that the above value of linear stiffness of the support is not only practically realizable but also large enough to make the support deflection negligible. In such a case, all the changes in the modal characteristics of the launch vehicle can be attributed to a shift in the nodal point location. It can be seen from Table 5 that, similarly to the uniform launch vehicle case, there is a significant change in the fundamental mode frequency for both positive and negative values of the error parameter, $e$, and is of the order of 9% for a 6% +ve error in the support position.

Furthermore, it can be seen that the same variation for −ve values of $e$ is of the order of 5%, indicating a reduced sensitivity in this case, a trend similar to the one observed for the uniform launch vehicle case. This shows not only that the fundamental mode is fairly sensitive to the support location, but also that the sensitivity increases with increase in the error. Furthermore, the influence of the +ve support shift is comparatively more substantial on the generalized mass and for negative values of $e$ the effect on $m$ is fairly small. In Figure 1 is shown the actual fundamental mode shape for both +ve and −ve values of the error parameter, which confirms the above observation that the mode shape changes are in fact more significant for the +ve error than those for the −ve error. It is also interesting to observe that the effect of shifting the support point, in both +ve and −ve error cases, on the fundamental mode shape is equivalent to an effective upward (or downward) rotation of the launch vehicle segment that will shift the nodal point to the new location. This fact can be verified for the case of $e = 0{\cdot}02$, where the actual free–free modal displacement of 0·03 is made zero by upward rotation of the launch vehicle segment up to the c.g. of the launch vehicle (∼35%); i.e., nearly as a rigid body with the c.g. as hinge point.

In the case of −ve values of $e$, the effect can be captured by an appropriate downward rotation of the same segment, but by a much smaller amount. This can be attributed to the fact that −ve error in the support location effectively increases the distance between the two support points and, thereby, reduces the overall changes in the displacement pattern due to this shift. The trends of variation indicated by the above results, which are presented in non-dimensional form, are also applicable to different classes of launch vehicle, because the nature of stiffness and mass distribution across various launch vehicle structures are similar to those considered in the present example. In fact, the results show that in terms of percentage changes in frequency for the same change in the non-dimensional error, the results for the generic launch vehicle are similar to those obtained for the non-dimensionalized uniform launch vehicle case, presented in Tables 2–4. However, in order that these conclusions can be further generalized and presented in a form that can be used for making accurate modal predictions, a detailed study of the normalized version of the launch vehicle structural configuration (given in Table 1) considered here is carried out, and the various results are presented in Table 6 and Figure 2.

Table 6

*Normalized Launch Vehicle Support Error Sensitivity*: $EI_0 = 5{\cdot}88$, $\rho A_0 = 1{\cdot}0$, *total mass* = 1·0, *total length* = 1·0

| First node (0·200) error results | | | Second node (0·679) error results | | |
|---|---|---|---|---|---|
| Error | Frequency, $\lambda$ | Mass, $m$ | Error | Frequency, $\lambda$ | Mass, $m$ |
| −0·05 | 4·885 | 0·0444 | −0·040 | 4·892 | 0·0308 |
| −0·04 | 4·917 | 0·0433 | −0·032 | 4·924 | 0·0323 |
| −0·03 | 4·944 | 0·0426 | −0·024 | 4·949 | 0·0343 |
| −0·02 | 4·965 | 0·0425 | −0·016 | 4·968 | 0·0370 |
| −0·01 | 4·979 | 0·0430 | −0·008 | 4·980 | 0·0430 |
| 0·00 | 4·984 | 0·0443 | −0·000 | 4·984 | 0·0445 |
| 0·01 | 4·979 | 0·0468 | 0·002 | 4·983 | 0·0546 |
| 0·02 | 4·962 | 0·0506 | 0·004 | 4·982 | 0·0469 |
| 0·03 | 4·933 | 0·05665 | 0·006 | 4·981 | 0·0482 |
| 0·04 | 4·892 | 0·0648 | 0·008 | 4·980 | 0·0495 |
| 0·05 | 4·837 | 0·0765 | 0·010 | 4·978 | 0·0509 |

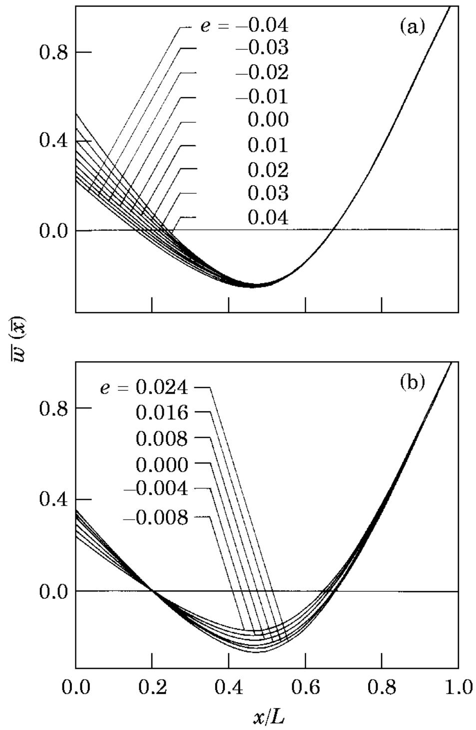


Figure 2. The normalized fundamental mode of the normalized generic launch vehicle for both +ve and −ve support errors, for (a) the first support point and (b) the second support point.

These results are condensed, and simple tools for predicting the free–free fundamental mode characteristics are developed in the next section.

## 5. MODAL PREDICTION TOOLS FOR GENERIC LAUNCH VEHICLES

The observations related to the sensitivity of the fundamental mode of a generic launch vehicle, made in the previous section, can be generalized in the form of simple expressions for a normalized launch vehicle that can be used for predicting the free–free mode from the supported results. The normalized launch vehicle is viewed as a geometry, the total mass and length of which are taken as unity, and the bending rigidity of the tip station is also taken as unity. The structural properties of all the segments are then derived in suitable proportions to the reference parameters. It may be mentioned here that such a normalized structural geometry is generally applicable to a large class of launch vehicles, and therefore the non-dimensional results arrived at for this case can be used for many practical launch vehicle geometries. In the present case, the geometry of Table 1 is normalized as mentioned above and the fundamental mode is extracted for support errors in both the front and the rear nodal point. In Table 6 are presented the results for the frequency parameter $\lambda$ and the modal mass parameter $m$ for the two cases, and in Figure 2 are presented typical mode shapes, for the reference normalized value of $EI_0 = 5{\cdot}88$, corresponding to the stiffest segment of the normalized launch vehicle. These results show that, for the range of error parameter considered, the frequency parameter varies quadratically for both +ve as well as −ve values of the error parameter $e$. The results also show that the variation for $\lambda$ for the second support point is nearly of the same nature as the first support, and therefore it is indeed possible to define a single expression for the predicted free frequency parameter $\lambda_0$, in terms of the measured frequency parameter $\lambda_m$, as

$$\lambda_0 = \lambda_m + 44{\cdot}7(e_1^2 + e_2^2) \tag{12}$$

where $e_1$ and $e_2$ are the non-dimensional errors at the first and second support points. It is seen from the above expression that for small errors it is possible to obtain the correct free frequency from experimentally measured data. The results for the modal mass, on the other hand, show that while the variation is predominantly quadratic, the actual magnitudes of variations are different for −ve errors from those observed for +ve support errors, and also different for both the first and the second support points. However, it is still possible to identify a common characteristic of quadratic variation for +ve errors and −ve errors, respectively, as

$$m_0 = m_m - 16{\cdot}0e_1^2 - 32{\cdot}0e_2^2 \qquad \text{for } e_1,\ e_2 > 0{\cdot}0, \tag{13}$$

$$m_0 = m_m + 0{\cdot}09e_1^2 + 0{\cdot}33e_2^2 \qquad \text{for } e_1,\ e_2 < 0{\cdot}0, \tag{14}$$

where $m_0$ is the predicted mass and $m_m$ is the measured mass. In addition to these relations, another important modal parameter is the modal displacement at the root station which can influence the servoelastic coupling when thrust vector control is used. In Figure 2 is shown the nature of variation of the root modal deflection as a function of the +ve as well as −ve error at both the first and second support points. It is seen that this parameter increases linearly with +ve error in the support location and decreases linearly with −ve error in support point, although by a smaller amount. The trend is similar for the first as well as the second support point, except that for the second support error, both +ve and −ve errors produce the same sensitivity. A set of simple expressions for the root modal deflection parameter $w_{er}$, which reflect the above observation, can be obtained as follows:

$$w_{er0} = w_{erm} + 5{\cdot}0e_1 + 3{\cdot}3e_2 \qquad \text{for } e_1 > 0{\cdot}0, \tag{15}$$

$$w_{er0} = w_{erm} + 2{\cdot}5e_1 + 3{\cdot}3e_2 \qquad \text{for } e_1 < 0{\cdot}0. \tag{16}$$

It may be mentioned here that the constants appearing in equations (12)–(16) have been arrived at by a least squares fit for the data presented in Table 6 and Figure 2 and, further, that these five relations are considered to be very useful tools for predicting the true free–free fundamental mode of a generic launch vehicle from the data measured from the ground resonance tests on the two point supported structure.

## 6. CONCLUSIONS

The problem of predicting the correct fundamental mode characteristics of a free–free normalized generic launch vehicle structure from the vibration test results carried out on the corresponding incorrectly supported structure is undertaken. Starting from elementary beam theory, the free vibration problem of the support structure is formulated and the results for the first mode characteristics of a uniform beam, a generic launch vehicle and a normalized generic launch vehicle structure, in terms of the frequency and modal mass, are obtained for various cases of shift in the support location away from the exact nodal locations. The support is idealized as a very stiff linear spring and a very soft rotational spring, and the results show that the frequency changes significantly while modal mass changes substantially, with increasing errors at the correct locations of the support points. The results also show that the fundamental mode is not very sensitive to the presence of rotational restraint at the support point. Furthermore, the non-dimensionalized modal results from normalized launch vehicle analysis show promise for application to a large class of launch vehicle structures by suitably scaling the frequency results. A sensitivity analysis of these non-dimensional results for this purpose reveals that fundamental mode parameters are nearly quadratic functions of the support error. Simple algebraic expressions are evolved that can be used to predict these characteristics for the in-flight free–free mode of the generic launch vehicle. The author believes that the study provides an important empirical procedure for improving the accuracy of the fundamental mode predictions based on the ground vibration test results on two point supported structures.

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